Cambridge Assessment International Education

Cambridge IGCSE™

CANDIDATE NAME									
CENTRE NUMBER						CANDIDATE NUMBER				

0657794285

CAMBRIDGE INTERNATIONAL MATHEMATICS **0607/61**

Paper 6 Investigation and Modelling (Extended) **October/November 2023**

1 hour 40 minutes

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer both part **A** (Questions 1 to 5) and part **B** (Questions 6 to 10).
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods.
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working to communicate your mathematics clearly and precisely.

INFORMATION

- The total mark for this paper is 60.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages.

[Turn over

Answer **both** parts **A** and **B**.

A INVESTIGATION (QUESTIONS 1 to 5)

F-TYPE SEQUENCES (30 marks)

You are advised to spend no more than 50 minutes on this part.

This investigation explores patterns in a special type of sequence of positive integers.

In an *F-type sequence*:
- the first two terms are any two positive integers
- after the first two terms, each term is equal to the sum of the previous two terms.

1 Here is a table of the first nine terms of an F-type sequence.

The first term F_1 is 5.
The second term F_2 is 3.

F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9
5	3	8	11	19	30	49	79	128

In the table, $F_1 + F_2 = 5 + 3 = 8 = F_3$

$F_2 + F_3 = 3 + 8 = 11 = F_4$

$F_3 + F_4 = 8 + 11 = 19 = F_5$

and so on.

(a) Calculate the 10th term.

$F_{10} =$ ..... [2]

(b) (i) Complete the table.

F_2	$= 3$	$F_3 - F_1 = 3$
$F_2 + F_4$	$=$	$F_5 - F_1 = 14$
$F_2 + F_4 + F_6$	$=$	$F_7 - F_1 =$ .....
$F_2 + F_4 + F_6 + F_8$	$= 123$	$F_9 - F_1 =$ .....

[2]

(ii) Complete this statement.

$$F_2 + F_4 + F_6 + F_8 + F_{10} = F_{\ldots\ldots} - F_{\ldots\ldots}$$

[1]

(c) (i) Complete the table.

F_1	$= 5$	$F_2 + F_1 - F_2 = 5$
$F_1 + F_3$	$=$	$F_4 + F_1 - F_2 = 13$
$F_1 + F_3 + F_5$	$=$	$F_6 + F_1 - F_2 =$ .....
$F_1 + F_3 + F_5 + F_7$	$= 81$	$F_8 + F_1 - F_2 =$ .....

[2]

(ii) Complete this statement.

$$F_1 + F_3 + F_5 + F_7 + F_9 = F_{\ldots\ldots} + F_{\ldots\ldots} - F_{\ldots\ldots}$$

[1]

(d) Use your statements in **part (b)(ii)** and **part (c)(ii)**, and the definition of an F-type sequence, to show that

$$F_1 + F_2 + F_3 + F_4 + F_5 + F_6 + F_7 + F_8 + F_9 + F_{10} = F_{12} - F_2 .$$

[2]

(e) Use the statement in **part (d)** to complete this general statement.

$$F_1 + F_2 + F_3 + \cdots + F_n = F_{\ldots\ldots} - F_{\ldots\ldots}$$

[1]

2 In another F-type sequence the first term is 3 and the second term is 1.

(a) Complete the first five terms.

3, 1, ..., ..., ... [1]

(b) Is your statement in **Question 1(e)** correct for the sum of the first five terms in this sequence?

..... [3]

3 In another F-type sequence the 2nd term is 3 and the 12th term is 652.

(a) Use your answer to **Question 1(e)** to find the sum of the first 10 terms.

..... [2]

(b) The sum of the first 12 terms of this sequence is 1704.

Find the 10th term.

..... [3]

4 The Fibonacci sequence is a special F-type sequence.

The sequence starts 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, …

(a) Use this information and your answers to **Question 1(c)** to simplify this sum.

$$F_1 + F_3 + F_5 + \cdots + F_{2n-1}$$

..... [1]

(b) The 16th term in the Fibonacci sequence is 987.

Find the 8 different terms in the Fibonacci sequence that add up to 987.

..... [2]

5 The first four terms of an F-type sequence are a, b, c and d.

(a) There is a relationship between $c^2 - b^2$ and a simple combination of a and d.

Investigate this relationship by making up at least three numerical examples of F-type sequences. Write down this relationship.

..... [4]

(b) The first term of the F-type sequence is a and the second term is b.

(i) Write c and d in terms of a and b, in their simplest form.

$c =$ .....

$d =$ ..... [1]

(ii) Use algebra to show that the relationship in **part (a)** is correct.

[2]

The modelling task starts on the next page.

B MODELLING (QUESTIONS 6 to 10)

BIOLOGICAL AGE OF GOATS (30 marks)

You are advised to spend no more than 50 minutes on this part.

This task looks at the age, a, of a goat and its biological age, b, when compared to a human.

A goat's body ages more quickly than a human body.
At birth, a goat's age and its biological age are both 0.

When $a = 0$ then $b = 0$.

6 The life expectancy for a human is 73.5 years.
The life expectancy for a goat is 10.5 years, which matches the biological life expectancy of 73.5 years for a human.

When $a = 10.5$ then $b = 73.5$.

(a) Find a straight-line model, in its simplest form, for b in terms of a.
This is **Model M**.

..... [3]

(b) Sketch the graph of your model.

[2]

(c) A goat is 8 years old, so $a = 8$.

Find its biological age, b.

..... [1]

7 Goats age more quickly when young.
A goat that is 2 years old has a biological age of 24 years.
So, when $a = 2$, $b = 24$.

(a) Find a straight-line model for b in terms of a for $0 \leqslant a \leqslant 2$.

..... [1]

(b) After a goat reaches the age of 2 years, its biological age increases by 4 each year.

(i) Find its biological age, b, when $a = 10$.

..... [2]

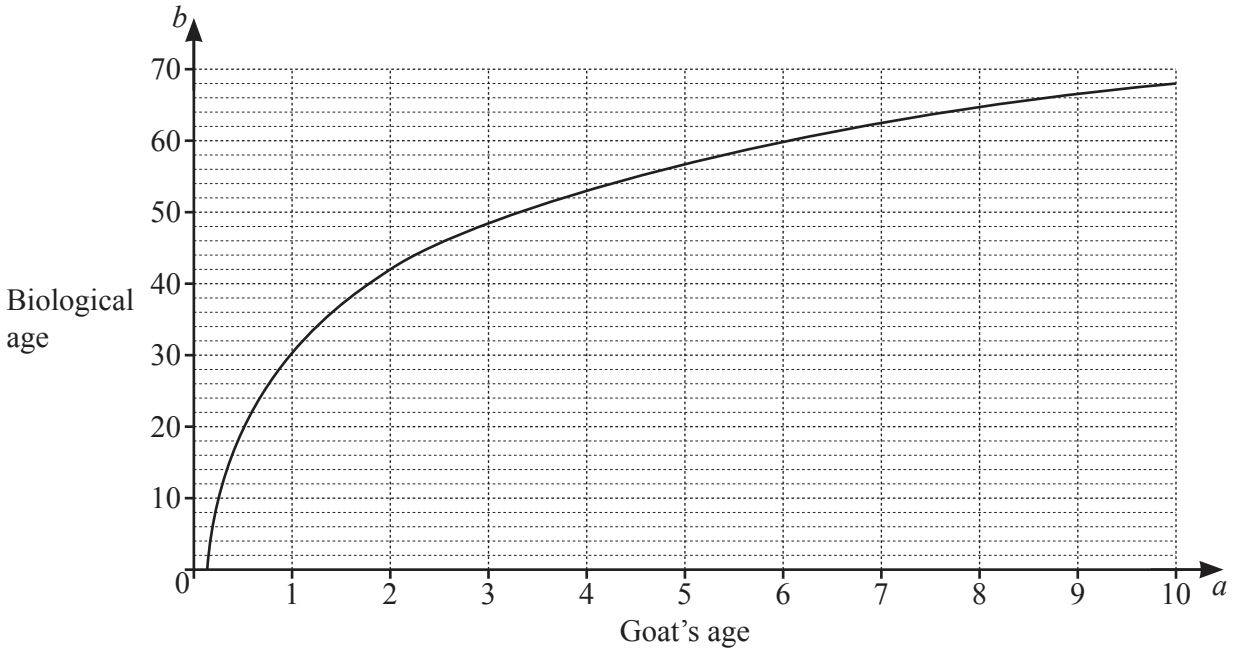
(ii) Find a straight-line model for b in terms of a for $a \geqslant 2$.
Write the model in its simplest form.
This is **Model N**.

..... [3]

(c) Sketch the graphs of your straight-line models in **part (a)** and **part (b)(ii)** on the axes on page 8.

[2]

8 The most recent research gives this graph for a model of b in terms of a.



(a) Use the graph to write down the biological age of a goat that is:

- 2 years old .....
- 10 years old. ..... [1]

(b) This model for the biological age is $b = g\log a + h$ where g and h are constants.

(i) Use your answers to **part (a)** to write down two equations in g and h.

.....

..... [1]

(ii) Use algebra to find g and h, correct to the nearest integer.
Write down the model.
This is **Model P**.

..... [3]

(c) Find the age, correct to one decimal place, of a goat whose biological age is 70.

..... [3]

9 A goat lives until it is 18 years old, which is old for a goat.

For each model calculate the biological age of the goat.
Write down whether each model is valid or not valid for this goat.

Model M in **Question 6(a)**

.....

Model N in **Question 7(b)(ii)**

.....

Model P in **Question 8(b)(ii)**

.....
[4]

Question 10 is printed on the next page.

10 Find the ages between which

biological age from **Model N** $<$ biological age from **Model P** $<$ biological age from **Model M**.

Between ..... and ..... [4]